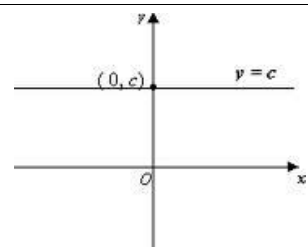
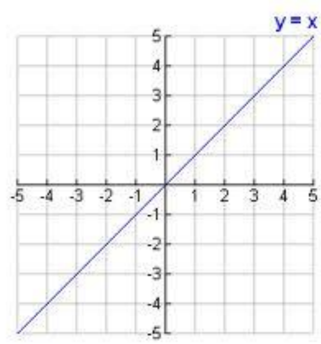
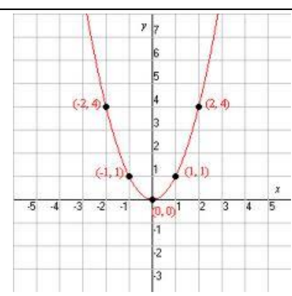
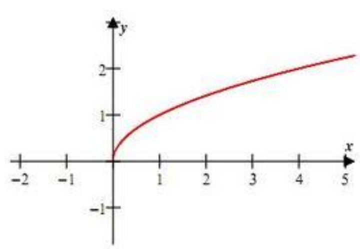
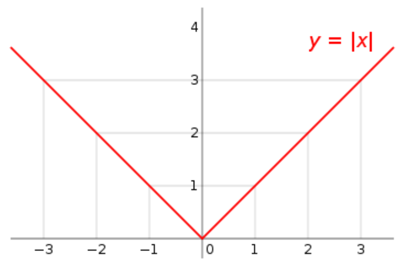
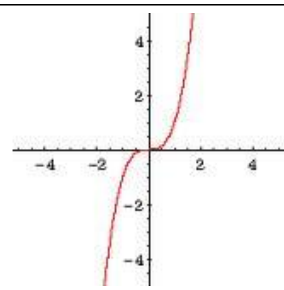
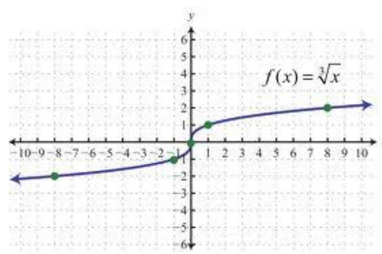
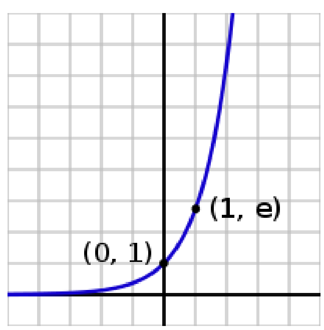
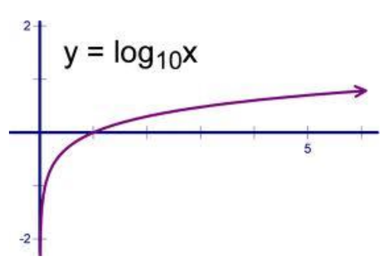
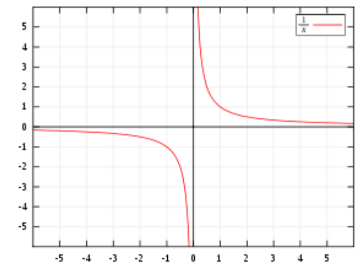
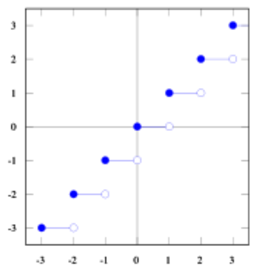
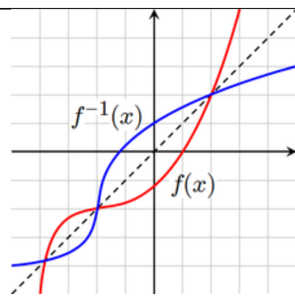
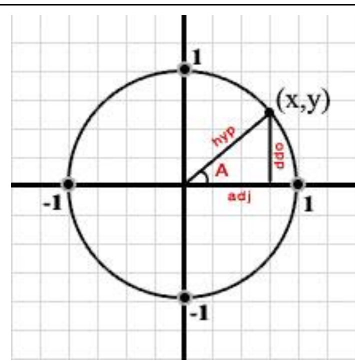
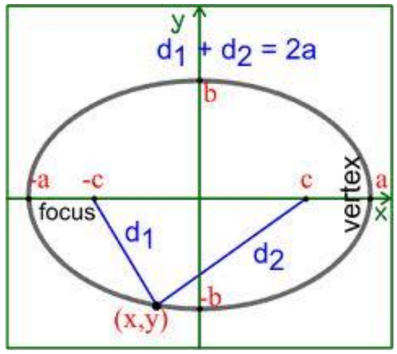
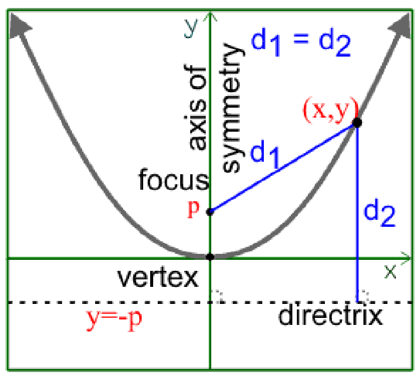
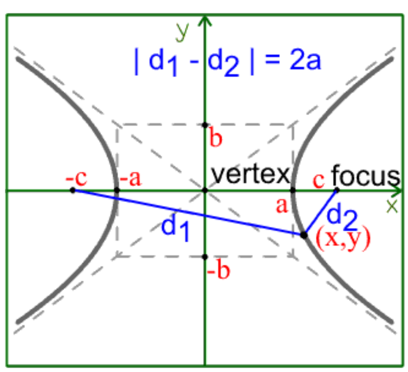
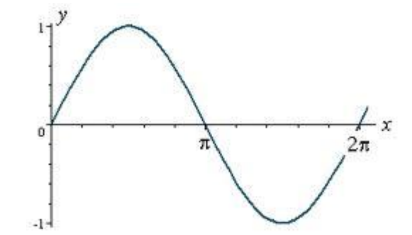
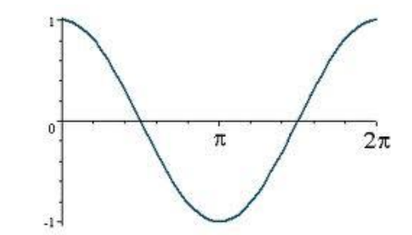
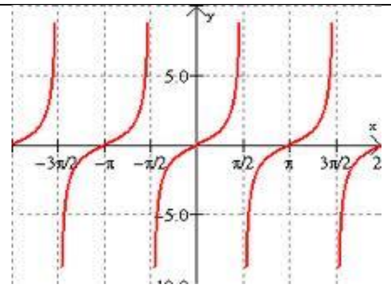
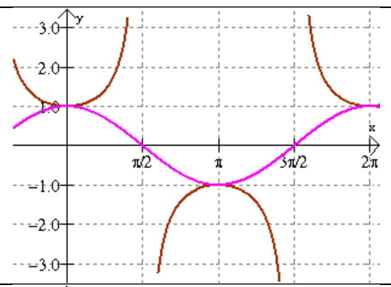
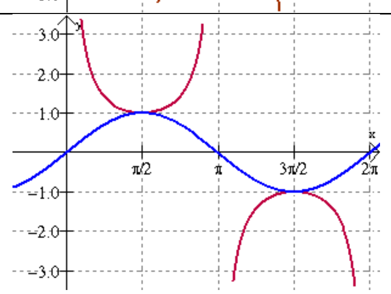
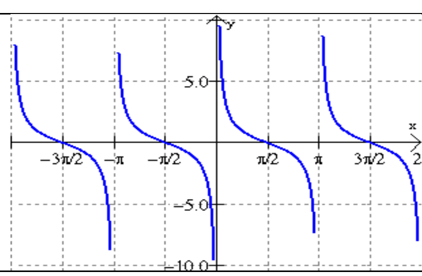
Harold's
Parent Functions
"Cheat Sheet"
24 November 2014

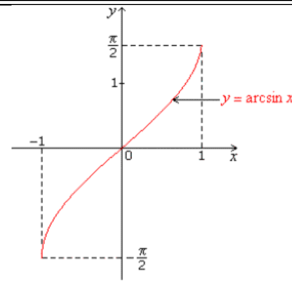
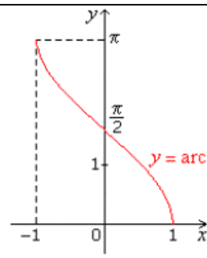
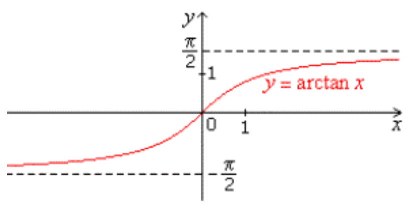
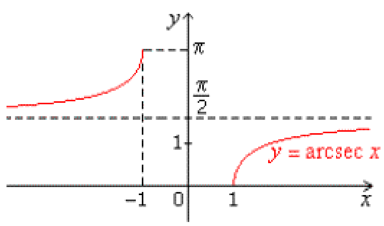
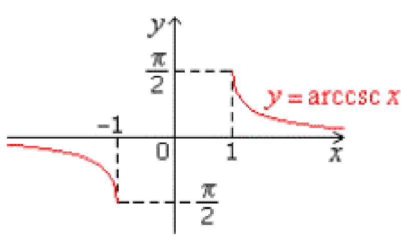
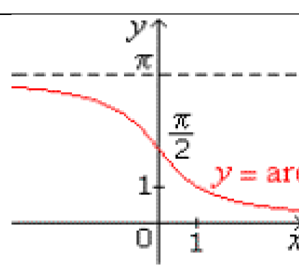
Function Name	Parent Function	Graph	Characteristics
Algebra			
Constant	$f(x) = c$		Domain: (-∞, ∞) Range: [c, c] Inverse Function: Undefined (asymptote) Restrictions: c is a real number Odd/Even: Even General Form: $Ay + B = 0$
Linear or Identity	$f(x) = x$		Domain: (-∞, ∞) Range: (-∞, ∞) Inverse Function: $g(x) = x$ Restrictions: m ≠ 0 Odd/Even: Odd General Forms: $Ax + By + C = 0$ $y = mx + b$ $y - y_0 = m(x - x_0)$
Quadratic or Square	$f(x) = x^2$		Domain: (-∞, ∞) Range: [0, ∞) Inverse Function: $g(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even General Form: $Ax^2 + By + Cx + D = 0$
Square Root	$f(x) = \sqrt{x}$		Domain: [0, ∞) Range: [0, ∞) Inverse Function: $g(x) = x^2$ Restrictions: $x \geq 0$ Odd/Even: Neither General Form: $f(x) = a\sqrt{b(x-h)} + k$

Function Name	Parent Function	Graph	Characteristics
Absolute Value	$f(x) = \|x\|$		Domain: (-∞, ∞) Range: [0, ∞) Inverse Function: $f(x) = x \; for \; x \geq 0$ Restrictions: $f(x) = \begin{cases} x, & if \; x \geq 0 \\ -x, & if \; x < 0 \end{cases}$ Odd/Even: Even General Form: $f(x) = a\|b(x-h)\| + k$
Cubic	$f(x) = x^3$		Domain: (-∞, ∞) Range: (-∞, ∞) Inverse Function: $g(x) = \sqrt[3]{x}$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a(b(x-h))^3 + k$
Cube Root	$f(x) = \sqrt[3]{x}$		Domain: (-∞, ∞) Range: (-∞, ∞) Inverse Function: $g(x) = x^3$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\sqrt[3]{b(x-h)} + k$
Exponential	$f(x) = 10^x$ *or* $f(x) = e^x$		Domain: (-∞, ∞) Range: (0, ∞) Inverse Function: $g(x) = \log x$ *or* $g(x) = \ln x$ Restrictions: None, x can be imaginary Odd/Even: Neither General Form: $f(x) = a\;10^{(b(x-h))} + k$
Logarithmic	$f(x) = \log x$ *or* $f(x) = \ln x$		Domain: (0, ∞) Range: (-∞, ∞) Inverse Function: $g(x) = 10^x$ *or* $g(x) = e^x$ Restrictions: x > 0 Odd/Even: Neither General Form: $f(x) = a\log(b(x-h)) + k$

Function Name	Parent Function	Graph	Characteristics
Reciprocal or Rational	$f(x) = \frac{1}{x}$		Domain: (-∞, 0) ∪ (0, ∞) Range: (-∞, 0) ∪ (0, ∞) Inverse Function: $g(x) = \frac{1}{x}$ Restrictions: x ≠ 0 Odd/Even: Odd General Form: $f(x) = a\left[\frac{b}{(x-h)}\right] + k$
Greatest Integer or Floor	$f(x) = [x]$		Domain: (-∞, ∞) Range: (-∞, ∞) whole numbers only Inverse Function: Undefined (asymptotic) Restrictions: Real numbers only Odd/Even: Neither General Form: $f(x) = a[b(x-h)] + k$
Inverse Functions	$If\ f(x) = y,\ then\ f^{-1}(y) = f^{-1}(f(x)) = x$		Domain of x → Domain of y Range of y → Range of x Inverse Function: By definition Restrictions: None Odd/Even: Odd General Form: $f(x) = a\,f(b(x-h)) + k$
Conic Sections			
Circle	$x^2 + y^2 = r^2$		Domain: $[-r+h,\ r+h]$ Range: $[-r+k,\ r+k]$ Inverse Function: Same as parent Restrictions: None Odd/Even: Both Focus : (h, k) General Forms: $(x-h)^2 + (y-k)^2 = r^2$ $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ $where\ A = C\ and\ B = 0$

Function Name	Parent Function	Graph	Characteristics
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$		Domain: $[-a+h,\ a+h]$ Range: $[-b+k,\ b+k]$ Inverse Function: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ Restrictions: None Odd/Even: Both Foci : $c^2 = a^2 - b^2$ General Forms: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ *where* $B^2-4AC<0$
Parabola	$y = ax^2$		Domain: (-∞, ∞) Range: $[k,\infty)$ or $(-\infty,k]$ Inverse Function: $g(x) = \sqrt{x}$ Restrictions: None Odd/Even: Even Vertex : (h,k) Focus : $(h,k+p)$ General Forms: $(x-h)^2 = 4p(y-k)$ $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ *where* $B^2-4AC=0$
Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$		Domain: (-∞, *-a+h*] ∩ [*a+h*, ∞) Range: (-∞, ∞) Inverse Function: $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ Restrictions: Domain is restricted Odd/Even: Both Foci : $c^2 = a^2 + b^2$ General Forms: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ *where* $B^2-4AC>0$

Function Name	Parent Function	Graph	Characteristics
Trigonometry			
Sine	$f(x) = sin\, x$		Domain: (-∞, ∞) Range: [-1, 1] Inverse Function: $g(x) = sin^{-1} x$ Restrictions: None Odd/Even: Odd General Form: $f(x) = a\ sin\left(b(x-h)\right) + k$
Cosine	$f(x) = cos\, x$		Domain: (-∞, ∞) Range: [-1, 1] Inverse Function: $g(x) = cos^{-1} x$ Restrictions: None Odd/Even: Even General Form: $f(x) = a\ cos\left(b(x-h)\right) + k$
Tangent	$f(x) = tan\, x$ $= \frac{sin\, x}{cos\, x}$		Domain: (-∞, ∞) except for $x = \frac{\pi}{2} \pm n\pi$ Range: (-∞, ∞) Inverse Function: $g(x) = tan^{-1} x$ Restrictions: Asymptotes at $x = \frac{\pi}{2} \pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, tan\left(b(x-h)\right) + k$
Secant	$f(x) = \sec x$ $= \frac{1}{cos\, x}$		Domain: (-∞, ∞) except for $x = \frac{\pi}{2} \pm n\pi$ Range: (-∞, -1] ∪ [1, ∞) Inverse Function: $g(x) = sec^{-1} x$ Restrictions: Range is bounded Odd/Even: Even General Form: $f(x) = a\ sec\left(b(x-h)\right) + k$
Cosecant	$f(x) = csc\, x$ $= \frac{1}{sin\, x}$		Domain: (-∞, ∞) except for $x = \pm n\pi$ Range: (-∞, -1] ∪ [1, ∞) Inverse Function: $g(x) = csc^{-1} x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\ csc\left(b(x-h)\right) + k$
Cotangent	$f(x) = cot\, x$ $= \frac{1}{tan\, x}$		Domain: (-∞, ∞) except for $x = \pm n\pi$ Range: (-∞, ∞) Inverse Function: $g(x) = cot^{-1} x$ Restrictions: Asymptotes at x = $\pm n\pi$ Odd/Even: Odd General Form: $f(x) = a\, cot\left(b(x-h)\right) + k$

Function Name	Parent Function	Graph	Characteristics
Arcsine	$f(x) = sin^{-1}x$		Domain: [-1, 1] Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or Quadrants I & IV Inverse Function: $g(x) = sin\,x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\,sin^{-1}\big(b(x-h)\big) + k$
Arccosine	$f(x) = cos^{-1}x$		Domain: [-1, 1] Range: $[0,\ \pi]$ or Quadrants I & II Inverse Function: $g(x) = cos\,x$ Restrictions: Range & Domain are bounded Odd/Even: None General Form: $f(x) = a\ cos^{-1}\big(b(x-h)\big) + k$
Arctangent	$f(x) = tan^{-1}x$		Domain: (-∞, ∞) Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ or Quadrants I & IV Inverse Function: $g(x) = tan\,x$ Restrictions: Range is bounded Odd/Even: Odd General Form: $f(x) = a\,tan^{-1}\big(b(x-h)\big) + k$
Arcsecant	$f(x) = sec^{-1}x$		Domain: (-∞, -1] ∪ [1, ∞) Range: $[0, \frac{\pi}{2})$ ∪ $(\frac{\pi}{2}, \pi]$ or Quadrants I & II Inverse Function: $g(x) = sec\,x$ Restrictions: Range & Domain are bounded Odd/Even: Neither General Form: $f(x) = a\,sec^{-1}\big(b(x-h)\big) + k$
Arccosecant	$f(x) = csc^{-1}x$		Domain: (-∞, -1] ∪ [1, ∞) Range: $[-\frac{\pi}{2}, 0)$ ∪ $(0, \frac{\pi}{2}]$ or Quadrants I & IV Inverse Function: $g(x) = csc\,x$ Restrictions: Range & Domain are bounded Odd/Even: Odd General Form: $f(x) = a\ csc^{-1}\big(b(x-h)\big) + k$
Arccotangent	$f(x) = cot^{-1}x$		Domain: (-∞, ∞) Range: $(0, \pi)$ or Quadrants I & II Inverse Function: $g(x) = cot\,x$ Restrictions: Range is bounded Odd/Even: Neither General Form: $f(x) = a\,cot^{-1}\big(b(x-h)\big) + k$

Function Name	Parent Function	Graph	Characteristics

Graphing Tips

All Functions

The Six Function "Levers"	*y = a f (b (x - h)) + k*	Graphing Tips
1) Move up/down ↕	k (Vertical translation)	"+" Moves it up
2) Move left/right ↔	h (Phase shift)	"+" Moves it right
3) Stretch up/down ↕	a (Amplitude)	Larger stretches it taller or make sit grow faster
4) Stretch left/right ↔	b (Frequency • 2π)	Larger stretches it wider
5) Flip about x-axis	a → –a	*f(x)* → *–f(x)* If *f(x)* = *–f(-x)* then odd function
6) Flip about y-axis	b → –b	*f(x)* → *f(-x)* If *f(x)* = *f(-x)* then even function

Trigonometric Functions

The Six Trig "Levers"	*y = a sin (b (x - h)) + k*	Graphing Tips	Notes
1) Move up/down ↕	k (Vertical translation)	$k = \frac{(\max + \min)}{2}$	If k = f(x) then x-axis is replaced by f(x)-axis
2) Move left/right ↔	h (Phase shift)	'+' shifts right	$\sin(x) = \cos(x - \pi/2)$
3) Stretch up/down ↕	a (Amplitude)	$a = \frac{(\max - \min)}{2}$	a is NOT peak-to-peak on y-axis
4) Stretch left/right ↔	b (Frequency • 2π)	$T = \frac{2\pi}{\lvert b \rvert} = \frac{1}{f}$	T = peak-to-peak on θ-axis T = π/b for tan (bx)
5) Flip about x-axis	a → –a	f(x) → –f(x)	Odd Function: $\sin(x) = -\sin(-x)$
6) Flip about y-axis	b → –b	f(x) → f(-x)	Even Function: $\cos(x) = \cos(-x)$